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Form Factors for $B \to \pi$ and $D \to \pi$ Transitions

Hai-Yang Cheng

Institute of Physics, Academia Sinica

Taipei, Taiwan 11529, Republic of China

Abstract

Armed with the information on the form factor $f_+^{D\pi}(0)$ inferred from recent CLEO measurements of SU(3)-breaking effects in charmed meson decays, we have studied the form factor $f_+^{B\pi}$. In the heavy quark limit, $f_+^{B\pi}(q^2)$ is related to $f_+^{D\pi}(q^2)$ in the kinematic region close to zero recoil. Assuming pole dominance for its q^2 dependence, $f_+^{B\pi}(0)$ is estimated to be ≈ 0.39. If the requirement of heavy quark symmetry is relaxed so that it applies only to soft pion emissions from the heavy meson, we find that $f_+^{B\pi}(0)$ is more likely of order $0.55 \sim 0.60$.

A reliable determination of the quark mixing matrix element V_{ub} from the semileptonic decay mode $\bar{B} \to \pi\ell\bar{\nu}$ ($\ell = e,\ \mu$) requires a knowledge of the $\bar{B} \to \pi$ transition form factor $f_+^{B\pi}$ at $q^2 = 0$. In the past, this form factor has been calculated using the nonrelativistic quark model [1], QCD sum rules [2], and heavy quark symmetry in synthesis with chiral symmetry [3-5]. A systematic analysis of the $1/m_b$ correction to the weak form factors $f_\pm^{B\pi}$ was recently studied in the framework of the heavy quark effective theory [6]. In the method of heavy quark symmetry, form factors $f_\pm^{B\pi}$ can be related in a model-independent way to the form factors $f_\pm^{D\pi}$ in the kinematic region close to zero recoil. In order to extract $f_+^{B\pi}(0)$ from the available experimental information of $f_+^{D\pi}(0)$, an extrapolation of the form factors from zero recoil to maximum recoil (i.e. $q^2 = 0$) has to be assumed. However, unlike the well measured form factor $f_+^{DK}(0)$, the present experimental data on $f_+^{D\pi}(0)$ are still plagued with large statistic and systematic errors. Fortunately, this situation was changed recently, Two new measured SU(3)-breaking effects in charm decays to be discussed later are very sensitive to the relative magnitude of the form factors $f_+^{D\pi}(0)$ and $f_+^{DK}(0)$. By fitting to the data, we found a best fit of $f_+^{D\pi}(0)/f_+^{DK}(0)$, and hence $f_+^{D\pi}(0)$. In this paper, we will study the form factor $f_+^{B\pi}(0)$ in two stages. In the first stage, heavy quark symmetry is applied so that $f_+^{B\pi}(q^2)$ is related to $f_+^{D\pi}(q^2)$ near zero recoil. In the second stage, the requirement of heavy quark symmetry is relaxed, namely it applies only to soft pion emissions from the heavy meson. We then make comments.

The matrix element of the $\bar{B} \to \pi$ transition is usually parametrized as

$$\langle\pi(p_\pi)|\bar{q}\gamma_\mu b|\bar{B}(v)\rangle = f_+^{B\pi}(q^2)(m_B v + p_\pi)_\mu + f_-^{B\pi}(q^2)(m_B v - p_\pi)_\mu, \tag{1}$$

or equivalently,

$$\langle\pi(p_\pi)|\bar{q}\gamma_\mu b|\bar{B}(v)\rangle = f_1^{B\pi}(q^2)(m_B v + p_\pi)_\mu + \frac{m_B^2 - m_\pi^2}{q^2} q_\mu [f_0^{B\pi}(q^2) - f_1^{B\pi}(q^2)], \tag{2}$$

where $q = m_B v - p_\pi$, and the form factors $f_\pm^{B\pi}$ and $f_{0,1}^{B\pi}$ are related by

$$f_1^{B\pi}(q^2) = f_+^{B\pi}(q^2), \quad f_0^{B\pi}(q^2) = f_+^{B\pi}(q^2) + \frac{q^2}{m_B^2 - m_\pi^2} f_-^{B\pi}(q^2). \tag{3}$$

To avoid unphysical poles at $q^2 = 0$ in Eq.(2), one must have $f_0(0) = f_1(0)$. In the $m_b \to \infty$ limit, the matrix element $\langle\pi|\bar{q}\gamma_\mu h_b|\bar{B}\rangle$ scales as $\sqrt{m_B}$, where h_b is the velocity-dependent effective heavy quark field for the b quark. Since

$$\bar{q}\gamma_\mu b = c(\mu)\bar{q}\gamma_\mu h_b(\mu) \tag{4}$$

at the subtraction scale $\mu < m_b$, and the large logarithmic contribution to $c(\mu)$ has been evaluated in Ref.[7], it follows from Eqs.(1) and (2) that

$$
\begin{aligned}
(f_+ + f_-)^{B\pi}(q_B^2) &= C_{bc}\sqrt{\frac{m_D}{m_B}}(f_+ + f_-)^{D\pi}(q_D^2), \\
(f_+ - f_-)^{B\pi}(q_B^2) &= C_{bc}\sqrt{\frac{m_B}{m_D}}(f_+ - f_-)^{D\pi}(q_D^2), \\
C_{bc} &= \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{-6/25},
\end{aligned}
\tag{5}
$$

where $q_B^2 = (m_B v - q)^2$, $q_D^2 = (m_D v - q)^2$. It is easily seen that the relations (5), which are first derived in Ref.[8], are valid provided that p does not scale with $m_{c,b}$ or $v \cdot p << m_{c,b}$. Eqs.(5) lead to

$$
f_+^{B\pi}(q_B^2) = \frac{C_{bc}}{2\sqrt{m_B m_D}}\left[(m_B + m_D) f_+^{D\pi}(q_D^2) - (m_B - m_D) f_-^{D\pi}(q_D^2)\right], \tag{6}
$$

$$
f_-^{B\pi}(q_B^2) = \frac{C_{bc}}{2\sqrt{m_B m_D}}\left[-(m_B - m_D) f_+^{D\pi}(q_D^2) + (m_B + m_D) f_-^{D\pi}(q_D^2)\right]. \tag{7}
$$

Note that since $(f_+ + f_-)^{B\pi}$ scales as $1/\sqrt{m_B}$ and $(f_+ + f_-)^{D\pi}$ as $1/\sqrt{m_D}$, Eqs.(6) and (7) are sometimes further reduced to

$$
f_+^{B\pi}(q_B^2) \approx -f_-^{B\pi}(q_B^2) = C_{bc}\sqrt{\frac{m_B}{m_D}} f_+^{D\pi}(q_D^2), \tag{8}
$$

so that $f_+^{B\pi}$ is expressed solely in terms of the physically measurable quantity $f_+^{D\pi}$, where use of the heavy-quark-symmetry approximation $f_-^{D\pi}(q_D^2) \approx -f_+^{D\pi}(q_D^2)$ has been made.

Recently it has been demonstrated that the form factor $f_1(q^2)$ has a monopole behavior in the combined large N_c, heavy quark and chiral limits [5]:

$$
f_1(q^2) = \frac{f_1(0)}{1 - \frac{q^2}{m_1^2}}, \tag{9}
$$

where m_1 is the mass of the lowest-lying 1^- resonance that couples to the weak current. Such a behavior is also seen in many QCD sum rule calculations [2]. However, this single pole behavior does not hold for the form factor $f_0(q^2)$, as one can see from Eq.(3) that

$$
f_0^{B\pi}(q^2) \approx f_+^{B\pi}(q^2)\left(1 - \frac{q^2}{m_B^2 - m_\pi^2}\right) \tag{10}
$$

under the heavy-quark-symmetry relation $f_-^{B\pi}(q^2) \approx -f_+^{B\pi}(q^2)$. Hence, the q^2 dependence of f_0 is different from that of f_+ by an additional pole factor [9]. In fact, if we follow Ref.[1]

to assume that $f_0(q^2) = f_0(0)/[1-(q^2/m_0^2)]$ with m_0 being the 0^+ pole mass, then it is easily seen that

$$f_-^{B(D)\pi}(0) = \left(m^2_{B(D)\pi} - m^2_\pi\right)\left(\frac{1}{m_0^2} - \frac{1}{m_1^2}\right) f_+^{B(D)\pi}(0). \tag{11}$$

Using $m_0 = 2.47$ (5.99) GeV and $m_1 = 2.01$ (5.32) GeV [1] for the form factors $f_{0,1}$ in $D-\pi$ ($\bar{B}-\pi$) transition, we find from Eq.(11) that

$$f_-^{B\pi}(0) = -0.21 f_+^{B\pi}(0), \qquad f_-^{D\pi}(0) = -0.29 f_+^{D\pi}(0), \tag{12}$$

which are substantially different from heavy-qaurk-symmetry expectations. From Eqs.(8) and (9) we find

$$f_+^{B\pi}(q^2_m) \;=\; 1.85 f_+^{D\pi}(q^2_m), \tag{13}$$

$$f_+^{B\pi}(0) \;=\; 0.47 f_+^{D\pi}(0), \tag{14}$$

where q^2_m is the momentum transfer squared at zero recoil and it is understood to be $(m_B - m_\pi)^2$ for $\bar{B}-\pi$ transition and $(m_D - m_\pi)^2$ for $D-\pi$ transition.

Presently, there are only two available experimental information on the form factor $f_+^{D\pi}(0)$. An earlier measurement of the Cabibbo-suppressed decay $D^0 \to \pi^-\ell^+\nu$ by Mark III yields $\left|f_+^{D\pi}(0)/f_+^{DK}(0)\right| = 1.0^{+0.6}_{-0.3} \pm 0.1$ [10,11], while a very recent CLEO-II measurement of $D^+ \to \pi^0\ell^+\nu$ gives $\left|f_+^{D\pi}(0)/f_+^{DK}(0)\right| = 1.29 \pm 0.21 \pm 0.11$ [12]. Though the latter perfers a larger $f_+^{D\pi}(0)$ over $f_+^{DK}(0)$, its error is still very large. Fortunately, a better determination of the ratio $f_+^{D\pi}(0)/f_+^{DK}(0)$ can be inferred from the recent CLEO measurements of the decay rates of $D^+ \to \pi^+\pi^0$ and $D^0 \to K^+\pi^-$ [13], which give the values of the ratios

$$R_1 \;=\; 2\left|\frac{V_{cs}}{V_{cd}}\right|^2 \frac{\Gamma(D^+ \to \pi^+\pi^0)}{\Gamma(D^+ \to \bar{K}^0\pi^+)} = 3.29 \pm 1.16\,, \tag{15}$$

$$R_2 \;=\; \left|\frac{V^*_{cs}V_{ud}}{V^*_{cd}V_{us}}\right|^2 \frac{\Gamma(D^0 \to K^+\pi^-)}{\Gamma(D^0 \to K^-\pi^+)} = 2.92 \pm 0.95 \pm 0.95\,. \tag{16}$$

Naively both ratios are expected to be unity if SU(3) is a good symmetry. The experimental values (15) and (16) thus appear quite striking at first glance. We have shown recently that such large SU(3) violations in R_1 and R_2 can be accounted for by the accumulations of several small SU(3)-breaking effects [14]. Crucial to our analysis is the relative magnitude of the form factors $f_+^{D\pi}(0) > f_+^{DK}(0)$, a necessary ingredient for obtaining the correct values

of R_1 and R_2. A fit of the large-N_c factorization calculation [14] to R_1 yields [1]

$$f_+^{D\pi}(0)/f_+^{DK}(0) \approx 1.09\,. \tag{17}$$

Using the average value [11]

$$f_+^{DK}(0) = 0.76 \pm 0.02 \tag{18}$$

extracted from the recent measurements of $D \to K\ell\bar{\nu}$ by CLEO, E687 and E691, we find [2]

$$f_+^{D\pi}(0) \approx 0.83\,. \tag{19}$$

Substituting (19) into (14) yields [3]

$$f_+^{B\pi}(0) \approx 0.39\,. \tag{20}$$

Thus far we have determined the form factor $f_+^{B\pi}$ from $f_+^{D\pi}$ via the heavy quark symmetry relation (8). It is known that, within the framework of chiral perturbation theory which incorporates both chiral and heavy quark symmetries [3,15], the form factor $f_+^{B\pi}$ near zero recoil is completely fixed by decay constants and the coupling constant $g_{B^*B\pi}$. From the heavy-meson chiral perturbation theory given in Refs.[3,15], we obtain ($f_\pi = 132$ MeV)

$$f_+^{B\pi}(q_B^2) + f_-^{B\pi}(q_B^2) = \frac{f_B}{f_\pi} \tag{21}$$

in the small $v \cdot p_\pi$ limit. Owing to the near degeneracy of the B^* and B masses, it also becomes necessary to take into account the B^* pole effects, which are [4]

$$\begin{aligned} f_+^{B\pi}(q_B^2) + f_-^{B\pi}(q_B^2) &= -\frac{f_{B^*}}{f_\pi}\,\frac{g v\cdot p_\pi}{v\cdot p_\pi + \Delta_B}\sqrt{\frac{m_B}{m_{B^*}}}, && (22)\\ f_+^{B\pi}(q_B^2) - f_-^{B\pi}(q_B^2) &= \frac{f_{B^*}}{f_\pi}\,\frac{g m_{B^*}^2/m_B}{v\cdot p_\pi + \Delta_B}\sqrt{\frac{m_B}{m_{B^*}}} && (23)\end{aligned}$$

[1] A determination of the ratio $f_+^{D\pi}(0)/f_+^{DK}(0)$ from R_2 is contaminated by the presence of the W-exchange diagrams and by possible final-state interactions.

[2] What we have done here is opposite to the procedure in Ref.[14]. There, the value $f_+^{D\pi}(0) \approx 0.83$ is first obtained by fitting the factorization calculation to the measured decay rates of $D^+ \to \pi^+\pi^0$ [although this value is obtained in Ref.[14] by assuming a monopole behavior for $f_0(q^2)$, the result remains the same if one considers the q^2 dependence of $f_+(q^2)$ instead of $f_0(q^2)$.] When combining with the experimental average value of $f_+^{DK}(0)$ given by (18), it then implies Eq.(17). For a comparsion, the values $f_+^{D\pi}(0) = 0.69$ and $f_+^{DK}(0) = 0.76$ are obtained in Ref.[1]. If these results were used in calculation, one would have obtained $R_1 = 1.4$, in disagreement with data.

[3] Baur, Stech and Wirbel [1] obtained $f_+^{B\pi}(0) = 0.333$.

[4] The sign of Eqs.(21-27) is opposite to that derived by M. Wise [3].

in the soft pion limit, where $\Delta_B = m_{B^*} - m_B$, g is the coupling constant in the heavy meson chiral Lagrangian [3,15], and we have neglected terms of order m_π/m_B, It follows from (22) and (23) that

$$f_+^{B\pi}(q_B^2) = \frac{f_B}{2f_\pi}\left(\frac{f_{B^*}}{f_B}\,\frac{g m_{B^*}\sqrt{m_{B^*}/m_B}}{v\cdot p_\pi + \Delta_B} + 1\right), \tag{24}$$

$$f_-^{B\pi}(q_B^2) = -\frac{f_B}{2f_\pi}\left(\frac{f_{B^*}}{f_B}\,\frac{g m_{B^*}\sqrt{m_{B^*}/m_B}}{v\cdot p_\pi + \Delta_B} - 1\right). \tag{25}$$

Likewise,

$$f_+^{D\pi}(q_D^2) = \frac{f_D}{2f_\pi}\left(\frac{f_{D^*}}{f_D}\,\frac{g m_{D^*}\sqrt{m_{D^*}/m_D}}{v\cdot p_\pi + \Delta_D} + 1\right), \tag{26}$$

$$f_-^{D\pi}(q_D^2) = -\frac{f_D}{2f_\pi}\left(\frac{f_{D^*}}{f_D}\,\frac{g m_{D^*}\sqrt{m_{D^*}/m_D}}{v\cdot p_\pi + \Delta_D} - 1\right). \tag{27}$$

It should be stressed that in the derivation of (24-27), heavy quark symmetry has been applied only to the soft pion emissions from any ground-state heavy meson so that the soft pion interaction with the heavy meson is described by a single coupling constant g. In the heavy quark limit where $\Delta_D = \Delta_B = 0$, $f_{D^*}/f_D = 1$, $f_{B^*}/f_B = 1$, $f_B/f_D = C_{bc}\sqrt{m_D/m_B}$ [7], it is easily seen that Eqs.(6) and (7) follow from Eqs.(24-27). Since

$$v\cdot p_\pi + \Delta_B = \frac{m_{B^*}}{2}\left(1 - \frac{q_B^2}{m_{B^*}^2}\right), \tag{28}$$

it is evident that, when q_B^2 is close to q_m^2, the form factor is single pole dominated. It has been argued that [16], beyond the soft pion limit, the relations (24-27) are still valid except that they must be multiplied by a factor of $(1-\alpha v\cdot p_\pi/\Lambda_\chi)$, with $\Lambda_\chi \sim 1$ GeV being a chiral symmetry breaking scale. The fact that the pole behavior shown by Eq.(9) is seen in many QCD sum rule calculations [2] over a large range of q^2 implies that $\alpha \approx 0$.

Unfortunately, since our present knowledge about the decay constants and in particular the coupling constant g is uncertain, we cannot predict the form factors $f_+^{B\pi}(0)$ and $f_+^{D\pi}(0)$ reliably through the above relations. Nevertheless, we can still learn something about $f_+^{B\pi}(0)$ based on the aforementioned value of $f_+^{D\pi}(0)$, lattice and QCD-sum-rule calculations for decay constants. For the purpose of illustration, we will take the central values of lattice calculations: $f_B = 187$ MeV [17], $f_D = 208$ MeV [17], $f_{D^*}/f_D \approx 1.16 f_{B^*}/f_B$ [18], and

QCD sum rule result $f_{B^*}/f_B \approx 1.1$ [19]. Assuming a monopole behavior for the form factor $f_+^{D\pi}(q^2)$, we find from Eq.(26)

$$g = 0.32\,, \tag{29}$$

which is substantially smaller than what naively expected from the chiral quark model [15]: $g = 0.75$. Substituting (29) into (24) yields

$$f_+^{B\pi}(0) = \ 0.53\,. \tag{30}$$

One can see from Eqs.(24) and (26) that a smaller $f_+^{B\pi}(0)$ requires a smaller strong coupling g, and hence a larger ratio of f_{D^*}/f_D, which is taken to be 1.3 in the above example. In general, the form factor $f_+^{B\pi}(0)$ is larger than 0.5 , to be compared with the value 0.39 inferred from Eq.(8) and the range $0.2 \sim 0.3$ obtained in QCD sum rule calculation [2]. As emphasized before, the chiral relations (24-27) are more general than Eqs.(6,7) since the requirement of heavy quark symmetry is relaxed in the former: it applies only to soft pion emissions from the heavy meson. Thus we believe that $f_+^{B\pi}(0)$ is more likely of order $0.55 \sim 0.60$. This should be checked soon by lattice calculation.

To conclude, using the value $f_+^{D\pi}(0) \approx 0.83$ inferred from recent CLEO measurements of SU(3) breaking effects in charm decays in conjunction with experimental results for $f_+^{DK}(0)$, we have studied the form factor $f_+^{B\pi}(0)$. We find that it is ≈ 0.39 in the heavy quark limit and of order $0.55 \sim 0.60$ when heavy quark symmetry is applied only to soft-pion $B^*B\pi$ and $D^*D\pi$ couplings.

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